# Soluzioni 7-Am3

Dott. Laura Di Gregorio

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**1.**

$$\begin{aligned}\int_0^\pi \int_0^1 (x\sin y - x^2 y)\ dx\,dy &= \int_0^\pi \left[\frac{x^2}{2}\sin y - \frac{x^3}{3}y\right]_0^1 dy\\ &= \int_0^\pi \left(\frac{1}{2}\sin y - \frac{1}{3}y\right) dy = 1 - \frac{\pi^2}{6}.\end{aligned}$$

D'altra parte

$$\begin{aligned}\int_0^1 \int_0^\pi (x\sin y - x^2 y)\ dy\,dx &= \int_0^1 \left[-x\cos y - x^2\frac{y^2}{2}\right]_0^\pi dx\\ &= \int_0^1 \left(x - x^2\frac{\pi^2}{2} + x\right) dx = 1 - \frac{\pi^2}{6}.\end{aligned}$$

**2.** Scegliendo la partizione in quadratini di lato

$$\delta = \frac{1}{n}$$

otteniamo

$$\lim_{n\to\infty} \sum_{i,j=1}^{n} \left(\frac{i}{n}\,\frac{j}{n}\right)\frac{1}{n^2} = \lim_{n\to\infty} \frac{1}{n^4}\,\frac{n(n+1)}{2}\,\frac{n(n+1)}{2} = \frac{1}{4}.$$