Science News Online

Week of March 10, 2007; Vol. 171, No. 10

Functional Family: Mock theta mystery solved

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A pair of mathematicians has solved a problem that had tantalized number-theory researchers for more than 8 decades. It is the so-called final problem of the legendary Indian mathematical genius Srinivasa Ramanujan.

In the years before his death in 1920, Ramanujan studied theta functions, which are numerical relationships that show special symmetries. On his deathbed, Ramanujan wrote a letter to his British collaborator G. H. Hardy, in which he listed 17 complicated formulas for new functions. He called them mock theta functions because they had some properties similar to those of theta functions.

The first few pages of Ramanujan's letter were lost, and the surviving portion gives little indication of why Ramanujan grouped these functions. Since that time, the mock theta functions have cropped up in a surprising array of fields, including number theory, probability theory, and statistical mechanics. Yet mathematicians have puzzled over just what the 17 mock theta functions have in common.

"The mock theta functions are like beautiful butterflies that Ramanujan happened to find," says Freeman Dyson, an emeritus professor at the Institute for Advanced Study in Princeton, N.J. "But if you're a scientist, you want more—you want a theory of evolution, a framework of ideas to fit the butterflies in."

Now, Ken Ono and Kathrin Bringmann, mathematicians at the University of Wisconsin–Madison, have supplied that theory. They figured out a definition of mock theta functions that covers all of Ramanujan's examples and shows how to build infinitely more such functions.

"I didn't really hope to see someone actually do this," says George Andrews of Pennsylvania State University in University Park, who had called the description of the mock theta functions one of the hardest math problems for the new millennium. Ono and Bringmann's accomplishment is "absolutely stunning," he concludes.

The reason that mathematicians have had trouble figuring out what the mock theta functions are, Ono says, is that in a certain sense, the functions are missing a piece. Building on 2002 work by Dutch mathematician Sander Zwegers, then at Utrecht University, Ono and Bringmann have shown that when certain functions are added to each of the mock theta functions, the results are highly symmetric expressions known as harmonic Maass forms.

The researchers report their findings in the March 6 *Proceedings of the National Academy of Sciences*. In two additional papers, they use their theory to prove longstanding conjectures about properties of the mock theta functions.

The new theory is likely to be valuable in many fields, Andrews says. "Whenever a mathematical subject is developed deeply, applications seem to crawl out of the woodwork," he notes.

The new work relies on contemporary mathematics that could not have been known to Ramanujan, says Bruce Berndt of the University of Illinois at Urbana-Champaign. "The task still remains to figure out what Ramanujan's ideas were," he says. "He had a viewpoint which we are still missing."

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http://www.sciencenews.org/articles/20070310/fob7.asp

From *Science News*, Vol. 171, No. 10, March 10, 2007, p. 149.