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D'Anna, Marco (I-CATN-MI); **Fontana, Marco** (I-ROME3)
The amalgamated duplication of a ring along a multiplicative-canonical ideal. (English summary)
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Let R be a commutative ring and M be an R-module. The idealization $R(+)M$ (also called the trivial extension), introduced by Nagata in 1956, is a ring where the module M can be viewed as an ideal such that its square is (0). A similar general construction, introduced recently by D'Anna and Fontana [see J. Algebra Appl. **6** (2007), no. 3, 443–459; MR2337762], is called the amalgamated duplication of a ring R along an ideal I and is denoted by $R \bowtie I$. When $I^2 = 0$, the new construction $R \bowtie I$ coincides with Nagata's idealization $R(+)I$. More precisely, the amalgamated duplication of R along an ideal I is a ring that is defined as the following subring of $R \times R$:

$$R \bowtie I = \{(r, r + i) |\ r \in R,\ i \in I\}.$$

On the other hand, the notion of multiplicative canonical ideal was introduced in the integral domain case by W. J. Heinzer, J. A. Huckaba and I. J. Papick [Comm. Algebra **26** (1998), no. 9, 3021–3043; MR1635902 (99h:13024)], and it can be easily extended to any commutative ring: a regular ideal I of a ring R is a multiplicative-canonical ideal of R if each regular fractional ideal J of R is I-reflexive, i.e., $J = (I\colon (I\colon J)) \cong \mathrm{Hom}_R(\mathrm{Hom}_R(J, I), I)$.

In this paper the authors study the properties of $R \bowtie I$ when I is a multiplicative canonical ideal of R.

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References

1. Bazzoni, S., Divisorial domains, *Forum Math.* **12** (2000), 397–419. MR1763897 (2001i:13031)
2. Bazzoni, S. and Salce, L., Warfield domains, *J. Algebra* **185** (1996), 836–868. MR1419726 (97j:13014)
3. Bruns, W. and Herzog, J., *Cohen–Macaulay Rings*, Cambridge Stud. Adv. Math. **39,** Cambridge University Press, Cambridge, 1993. MR1251956 (95h:13020)
4. D'Anna, M., A construction of Gorenstein rings, *J. Algebra* **306** (2006), 507–519. MR2271349 (2007j:13024)
5. D'Anna, M. and Fontana, M., An amalgamated duplication of a ring along an ideal: the basic properties, to appear in *J. Algebra Appl.* cf. MR 2008i:13002
6. Fossum, R., Commutative extensions by canonical modules are Gorenstein rings, *Proc. Amer. Math. Soc.* **40** (1973), 395–400. MR0318139 (47 #6688)
7. Fossum, R. M., Griffith, P. A. and Reiten, I., *Trivial Extensions of Abelian Categories*, Springer, Berlin–Heidelberg, 1975. MR0389981 (52 #10810)

8. Fuchs, L. and Salce, L., *Modules over Non-Noetherian Domains*, Math. Surveys Monogr. **84,** Amer. Math. Soc., Providence, RI, 2001. MR1794715 (2001i:13002)
9. Heinzer, W. J., Huckaba, J. A. and Papick, I. J., $\mathfrak{m}$-canonical ideals in integral domains, *Comm. Algebra* **26** (1998), 3021–3043. MR1635902 (99h:13024)
10. Huckaba, J. A., *Commutative Rings with Zero Divisors*, Monogr. Textbooks Pure Appl. Math. **117,** Marcel Dekker, New York, 1988. MR0938741 (89e:13001)
11. Kunz, E., Beispiel: Die kanonische Idealklasse eines eindimensionalen Cohen–Macaulay-Rings, in *Der kanonische Modul eines Cohen–Macaulay-Rings* (*Sem. Lokale Kohomologietheorie von Grothendieck*, *Univ. Regensburg*, *Regensburg*, 1970/1971), Lecture Notes in Math. **238,** pp. 17–24, 103, Springer, Berlin–Heidelberg, 1971. MR0476727 (57 #16285)
12. Maimani, H. R. and Yassemi, S., Zero-divisor graphs of amalgamated duplication of a ring along an ideal, *Preprint*, 2006. cf. MR 2008j:13007
13. Matlis, E., 1-*dimensional Cohen–Macaulay Rings*, Springer, Berlin–Heidelberg, 1973. MR0357391 (50 #9859)
14. Nagata, M., *Local Rings*, Interscience Tracts in Pure and Applied Mathematics **13,** Interscience-Wiley, New York–London, 1962. MR0155856 (27 #5790)
15. Olberding, B., Stability, duality, 2-generated ideals and a canonical decomposition of modules, *Rend. Sem. Mat. Univ. Padova* **106** (2001), 261–290. MR1876223 (2002k:13042)
16. Reiten, I., The converse to a theorem of Sharp on Gorenstein modules, *Proc. Amer. Math. Soc.* **32** (1972), 417–420. MR0296067 (45 #5128)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.