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On inverse limits of Bézout domains. (English summary)

Abelian groups, rings, modules, and homological algebra, 59–66, *Lect. Notes Pure Appl. Math.*, 249, *Chapman & Hall/CRC, Boca Raton, FL*, 2006.

Let $\{\varphi_{mn}\colon A_m \to A_n \mid m \geq n\}$ be an inverse system of commutative domains indexed by $\mathbb{N}$ and let $A = \varprojlim A_n$. The authors study the following question: is A a Prüfer domain when A_n is a Prüfer domain $\forall n \in \mathbb{N}$? In a previous article [D. E. Dobbs and M. Fontana, Math. Scand. **88** (2001), no. 1, 17–40; MR1813518 (2001i:13032)] they got a positive answer for certain types of inverse systems. In the present paper, an example shows that this question has generally a negative answer, but in this example the homomorphisms φ_{mn} are not surjective. So, in the sequel, the surjectivity of φ_{mn} and its restriction to the unit groups is assumed, $\forall m,\ \forall n,\ m \geq n$. When A_n is Bézout $\forall n \in \mathbb{N}$, they prove that A is Prüfer if the following additional condition holds: each nonzero prime ideal of A belongs to $\bigcup_{n\in\mathbb{N}} \operatorname{Im}(\operatorname{Spec}(A_n) \to \operatorname{Spec}(A))$. But, without this additional assumption, N. I. Dubrovin already obtained a better result since he showed that A is Bézout in this case [Mat. Sb. (N.S.) **119(161)** (1982), no. 1, 89–95; MR0672411 (84h:13023)]. At the end of the paper, an inverse system $\{\varphi^*_{mn}\colon A^*_m \to A^*_n \mid m \geq n\}$ is constructed where A^*_n is obtained from A_n by a suitable CPI-extension. If $\mathcal{C}$ is a class of integral domains then $A^* = \varprojlim A^*_n \in \mathcal{C}$ if $A_n \in \mathcal{C},\ \forall n \in \mathbb{N}$, in the following cases: $\mathcal{C}$ is the class of all (i) Prüfer domains, (ii) Bézout domains, (iii) divided domains, (iv) locally divided domains.

Reviewed by *François Couchot*

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