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Fontana, Marco (I-ROME3); **Popescu, Nicolae** (R-AOS)

Universal property of the Kaplansky ideal transform and affineness of open subsets. (English. English summary)

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Let R be an integral domain with quotient field K, let I be an ideal of R, and let $\Omega(I)$ denote the Kaplansky transform of I: $\Omega(I) = \{x \in K \mid$ for each $a \in I$ there is an integer $n(a) \geq 1$ such that $a^{n(a)}x \in R\}$. In the non-Noetherian setting, the Kaplansky transform is more tractable than the more familiar Nagata transform (they coincide when I is finitely generated). In the paper under review, the authors find a universal property of the canonical embedding $R \subseteq \Omega(I)$. In order to state the main result of the paper, a little terminology is needed. Given an ideal I of R, the authors call a ring homomorphism $\alpha\colon R \to A$ an I-morphism if the induced (continuous) map $\alpha^*\colon \mathrm{Spec}(A) \to \mathrm{Spec}(R)$ (given by $\alpha^*(Q) = \alpha^{-1}(Q)$ for $Q \in \mathrm{Spec}(A)$) satisfies $\alpha^*(\mathrm{Spec}(A)) \subseteq D(I) = \{P \in \mathrm{Spec}(R) \mid P \not\supseteq I\}$. Denote by $K_R(I, A)$ the set of all I-morphisms from R to A. Then $K_R(I, -)$ defines a covariant functor from the category of rings to the category of sets. The authors' main result is then as follows: This functor is representable $\Leftrightarrow$ the canonical embedding $R \subseteq \Omega(I)$ is an I-morphism $\Leftrightarrow$ $I\Omega(I) = \Omega(I)$ $\Leftrightarrow$ $D(I)$ is an affine open subscheme of $\mathrm{Spec}(R)$. *Evan G. Houston* (1-NC3)

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