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Essential domains and two conjectures in dimension theory. (English. English summary)

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This paper handles two long-standing conjectures in dimension theory concerning the Krull dimension of the integer-valued polynomial rings and the links between Krull domains and Jaffard domains.

Given a domain R, let $\mathrm{Int}(R)$ denote the integer-valued polynomial ring. A well-known property is that $\dim(R[X]) - 1 \leq \dim(\mathrm{Int}(R))$. The question that still remains open is whether $\dim(\mathrm{Int}(R)) \leq \dim(R[X])$. It is noted that all examples conceived in the literature satisfy this inequality. On the other hand, Bouvier's conjecture states that finite-dimensional Krull domains need not be Jaffard domains. Since the Krull property is stable under adjunction of indeterminates, it is reported that the problem merely deflates to the existence of a Krull domain R with $1 + \dim(R) < \dim(R[X])$.

The author first enlarges the scope of study of Bouvier's conjecture by legitimately raising the following problem: Is every finite-dimensional (locally) essential domain Jaffard? Then he settles the first conjecture in the affirmative for locally essential domains, showing that each locally essential domain R satisfies $\dim(\mathrm{Int}(R)) = \dim(R[X])$. Finally, an example is constructed to prove that for any integer $r \geq 2$, there exists an r-dimensional essential Jaffard domain that is not locally essential. *Samir Bouchiba* (Meknès)

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