Am1c – Soluzioni Tutorato I

Limiti di funzioni

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Filippo Cavallari, Fabio Pusateri

Esercizio 1

(1) $\lim\limits_{x\to\infty}\dfrac{x^6-6x^5+4x+3}{x^3-4}=\lim\limits_{x\to\infty}\dfrac{x^6\left(1-\dfrac{6}{x}+\dfrac{4}{x^5}+\dfrac{3}{x^6}\right)}{x^3\left(1-\dfrac{4}{x^3}\right)}=\lim\limits_{x\to\infty}\dfrac{x^3\left(1-\dfrac{6}{x}+\dfrac{4}{x^5}+\dfrac{3}{x^6}\right)}{\left(1-\dfrac{4}{x^3}\right)}=\infty$

(2) $\lim\limits_{x\to\infty}\dfrac{x^5-7x^4+12x^3+4}{x^8+14x^5-80}=\lim\limits_{x\to\infty}\dfrac{x^5\left(1-\dfrac{7}{x}+\dfrac{12}{x^2}+\dfrac{4}{x^5}\right)}{x^8\left(1+\dfrac{14}{x^3}-\dfrac{80}{x^8}\right)}=\lim\limits_{x\to\infty}\dfrac{\left(1-\dfrac{7}{x}+\dfrac{12}{x^2}+\dfrac{4}{x^5}\right)}{x^3\left(1+\dfrac{14}{x^3}-\dfrac{80}{x^8}\right)}=0$

(3) $\lim\limits_{x\to\infty}\dfrac{12x^6-14x^3+24}{\left(6x^3+2\right)^2}=\lim\limits_{x\to\infty}\dfrac{12x^6-14x^3+24}{36x^6+24x^3+4}=\lim\limits_{x\to\infty}\dfrac{x^6\left(12-\dfrac{14}{x^3}+\dfrac{24}{x^6}\right)}{x^6\left(36+\dfrac{24}{x^3}+\dfrac{4}{x^6}\right)}=\lim\limits_{x\to\infty}\dfrac{\left(12-\dfrac{14}{x^3}+\dfrac{24}{x^6}\right)}{\left(36+\dfrac{24}{x^3}+\dfrac{4}{x^6}\right)}=\dfrac{1}{3}$

(4) $\lim\limits_{x\to\infty}\sqrt{x}\sin\dfrac{1}{x}=\lim\limits_{y\to0}\dfrac{\sin y}{\sqrt{y}}=\lim\limits_{y\to0}\dfrac{\sin y}{y}\sqrt{y}=0$

(5) $\lim\limits_{x\to0}\dfrac{\arcsin x}{x}=\lim\limits_{y\to0}\dfrac{y}{\sin y}=1$

(6) $\lim\limits_{x\to\pi}\dfrac{\sin x}{x-\pi}=\lim\limits_{x\to\pi}-\dfrac{\sin\left(x-\pi\right)}{x-\pi}=\lim\limits_{y\to0}-\dfrac{\sin y}{y}=-1$

(7) $\lim\limits_{x\to0}\dfrac{1-\cos x}{x^2}=\lim\limits_{x\to0}\left(\dfrac{1-\cos x}{x^2}\dfrac{1+\cos x}{1+\cos x}\right)=\lim\limits_{x\to0}\dfrac{1-\cos^2 x}{x^2\left(1+\cos x\right)}=\lim\limits_{x\to0}\dfrac{\sin^2 x}{x^2}\dfrac{1}{1+\cos x}=\dfrac{1}{2}$

(8) $\lim\limits_{x\to0}\dfrac{\arctan x}{x}=\lim\limits_{y\to0}\dfrac{y}{\tan y}=\lim\limits_{y\to0}\left(\dfrac{y}{\sin y}\cos y\right)=1$

(9) $\lim\limits_{x\to e}\dfrac{x-e}{\sqrt{x}-\sqrt{e}}=\lim\limits_{x\to e}\left(\dfrac{x-e}{\sqrt{x}-\sqrt{e}}\dfrac{\sqrt{x}+\sqrt{e}}{\sqrt{x}+\sqrt{e}}\right)=\lim\limits_{x\to e}\sqrt{x}+\sqrt{e}=2\sqrt{e}$

(10) $\lim\limits_{x\to0}\dfrac{\ln\left(1+x\right)}{x}=\lim\limits_{y\to\infty}y\ln\left(1+\dfrac{1}{y}\right)=\lim\limits_{y\to\infty}\ln\left(1+\dfrac{1}{y}\right)^y=1$

(11) $\lim\limits_{x\to4}\dfrac{x^4-4x^3-3x^2+14x-8}{x^6-4x^5+x^4-4x^3}=\lim\limits_{x\to4}\dfrac{\left(x^3-3x+2\right)\left(x-4\right)}{\left(x^5+x^3\right)\left(x-4\right)}=\dfrac{27}{544}$

(12) $\lim\limits_{x\to0^+}x^x=\lim\limits_{x\to0^+}e^{x\ln x}=e^0=1$

(13) $\lim\limits_{x\to0}\dfrac{x\sin x}{1-\cos x}=\lim\limits_{x\to0}\dfrac{x^2}{1-\cos x}\dfrac{\sin x}{x}=2$

(14) $\lim\limits_{x\to0}\dfrac{7^{11x}-1}{x}=\lim\limits_{x\to0}\dfrac{e^{x11\ln7}-1}{x}=\lim\limits_{x\to0}\dfrac{e^{x11\ln7}-1}{x11\ln7}11\ln7=11\ln7$

(15) $$\lim_{x\to 0}\frac{A^x-B^x}{x}=\lim_{x\to 0}\frac{B^x\left[\left(\frac{A}{B}\right)^x-1\right]}{x}=\lim_{x\to 0}\frac{B^x\left[e^{x\ln\frac{A}{B}}-1\right]}{x}=\lim_{x\to 0}\frac{B^x\left[e^{x\ln\frac{A}{B}}-1\right]}{x\ln\frac{A}{B}}\ln\frac{A}{B}=\ln\frac{A}{B}$$

(16) $$\lim_{x\to 0}\frac{\sqrt{3+x}-\sqrt{3-x}}{\sqrt{5+x}-\sqrt{5-x}}=\lim_{x\to 0}\frac{\sqrt{3+x}-\sqrt{3-x}}{\sqrt{5+x}-\sqrt{5-x}}\,\frac{\sqrt{3+x}+\sqrt{3-x}}{\sqrt{5+x}+\sqrt{5-x}}\,\frac{\sqrt{5+x}+\sqrt{5-x}}{\sqrt{3+x}+\sqrt{3-x}}=$$
$$=\lim_{x\to 0}\frac{\sqrt{5+x}+\sqrt{5-x}}{\sqrt{3+x}+\sqrt{3-x}}=\sqrt{\frac{5}{3}}$$

(17) $$\lim_{x\to 0}\frac{\ln\left(\tan^4 x+1\right)}{e^{2\sin^4 x}-1}=\lim_{x\to 0}\frac{\ln\left(\tan^4 x+1\right)}{\tan^4 x}\,\frac{2\sin^4 x}{e^{2\sin^4 x}-1}\,\frac{1}{2\cos^4 x}=\frac{1}{2}$$

(18) $$\lim_{x\to 0}\frac{\ln\sin^2 x^2}{\ln x^4}=\lim_{x\to 0}\frac{\ln\sin^2 x^2-\ln x^4+\ln x^4}{\ln x^4}=\lim_{x\to 0}\frac{\ln\frac{\sin^2 x^2}{x^4}+\ln x^4}{\ln x^4}=1$$

(19) $$\lim_{x\to 0}\left(\sin x^2\right)^{\frac{1}{\log_{17}x^2}}=\lim_{x\to 0}17^{\frac{\log_{17}\sin x^2}{\log_{17}x^2}}=17$$

(20) Dalle formule di addizione della tangente $\tan\left(\alpha+\beta\right)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$ si ottiene

$$\lim_{x\to 0}x\tan\left(xa+\arctan\frac{b}{x}\right)=\lim_{x\to 0}x\frac{\tan ax+\frac{b}{x}}{1-\frac{b}{x}\tan ax}=\lim_{x\to 0}\frac{x\tan ax+b}{1-ab\frac{\tan ax}{ax}}=\begin{cases}\dfrac{b}{1-ab} & ab\neq 1\\ \infty & ab=1\end{cases}$$

Esercizio 2 (1) Basta notare che, posto $f(x)=\dfrac{1}{\sin\frac{1}{x}}$, $f(x_n)\xrightarrow{n\to+\infty}\infty$ se $x_n=\dfrac{1}{n\pi}$ mentre $f(x_n)\xrightarrow{n\to+\infty}1$ se $x_n=\dfrac{1}{\frac{\pi}{2}+2n\pi}$.

(2) Posto $f(x)=\dfrac{|5-x|}{5-x}$ si ha $f(x_n)\xrightarrow{n\to+\infty}-1$ se $x_n=\dfrac{1}{n}+5$ mentre $f(x_n)\xrightarrow{n\to+\infty}1$ se $x_n=-\dfrac{1}{n}+5$. Infine si ha che $\lim_{x\to 5^+}\dfrac{|5-x|}{5-x}=-1$ e $\lim_{x\to 5^-}\dfrac{|5-x|}{5-x}=1$.